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ORIGINAL PAPER

Matching markets with farsighted couples

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Abstract
We adopt the notion of the farsighted stable set to determine which matchings are stable when agents are farsighted in matching markets with couples. We show that a singleton matching is a farsighted stable set if and only if the matching is stable. Thus, matchings that are stable with myopic agents remain stable when agents become farsighted. Examples of farsighted stable sets containing multiple non-stable matchings are provided for markets with and without stable matchings. For couples markets where the farsighted stable set does not exist, we propose the DEM farsighted stable set to predict the matchings that are stable when agents are farsighted.

1 Introduction

Early-labor markets have been an important application for market designers. In the classical setting of matching under preferences, there are two sides of the market, such as medical students applying for residency programs and hospitals that would like to fill their positions at their residency programs. Students submit their preference orderings over hospitals and hospitals submit their preference orderings over students. In this framework, clearinghouses are generally regarded as successful if they produce *stable* matchings in the sense that no agent has an incentive to change her assignment after being matched. The seminal paper of Gale and Shapley (1962), where the celebrated

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Deferred Acceptance algorithm is described, generated a vast literature on matching markets.[1]

An important problem in entry-level labor markets is motivated by the "externalities" generated by the presence of couples seeking for positions in the same labor market. Since members of a couple do not only care about their own assignment but also about their partner's assignment, it is more challenging to design matching mechanisms when couples are present. First, the existence of a stable matching is not guaranteed (Roth 1984). Second, even if a stable matching does exist, it is computationally difficult to find it (Ronn 1990). In the presence of couples that submit joint preference lists over pairs of hospitals, there may not exist a mechanism that yields a stable matching whenever one exists. Despite these difficulties, one of the most successful stories of market design is the medical residency programs (e. g. the National Resident Matching Program (NRMP) in the United States and the Scottish Foundation Allocation Scheme (SFAS) in Scotland).[2] In the NRMP, since the 1950s, the presence of couples looking for a job in the market have significantly increased making the matching problem more and more complex. To overcome this issue, the program was successfully redesigned by Roth and Peranson (1999).[3]

Stability has been a central problem in the literature of matching with couples. A stable matching always exists in couples markets when preferences are weakly responsive, i.e., when a unilateral improvement on acceptable positions for one partner's job is beneficial for the couple as well (Klaus and Klijn 2005; Klaus et al. 2009). Tello (2016) shows that in a model of matching with contracts for which the couples model studied in this paper is a particular case, bilateral substitutability, a domain broader than weak responsiveness, is a maximal domain for the existence of stable matchings. Kojima et al. (2013) show that in large markets with couples a stable matching exists when there are relatively few couples and preference lists are sufficiently short relative to market size. However, if the number of couples grows at a linear rate, no stable matching exists even in large random matching markets with couples (Ashlagi et al. 2014). Nguyen and Vohra (2018) provide a solution to this problem by perturbing slightly the capacity of hospitals.

The notion of stable matching is a myopic notion since agents do not anticipate that individual and coalitional deviations can be followed by subsequent deviations. Other solution concepts for two-sided matching problems like the von Neumann–Morgenstern (vNM) stable set (Ehlers 2007) or the reformulation of the vNM stable set in Herings et al. (2017), called CP vNM set,[4] are myopic notions based on the direct dominance relation and neglect the destabilizing effect of indirect dominance relations introduced by Harsanyi (1974) and Chwe (1994). Indirect dominance captures the idea that farsighted agents can anticipate the actions of other coalitions and consider the

[1] Roth and Sotomayor (1990) provide a comprehensive introduction on matching theory. Roth (2008) is a survey devoted to the Deferred Acceptance algorithm.

[2] Biró and Klijn (2013) give a detailed overview of matching with couples under preferences.

[3] Roth (2018) provides a recent survey discussing, from a market design perspective, the residency labor markets, their history and evolution.

[4] Contrary to the vNM stable set in Ehlers (2007), the CP vNM set satisfies coalitional sovereignty and is defined using path dominance instead of direct dominance.

end matching that their deviations may lead to. To the best of our knowledge, this is the first paper incorporating farsightedness in the literature on matchings with couples.

We adopt the notion of the farsighted stable set to determine which matchings are stable when agents are farsighted in couples markets. A set of matchings is a farsighted stable set if no matching inside the set is indirectly dominated by another matching in the set (internal stability) and any matching outside the set is indirectly dominated by some matching in the set (external stability). Mauleon et al. (2011) characterize the farsighted stable sets for marriage markets as all singletons that contain a stable matching.[5] Klaus et al. (2011) show that, for roommate markets, a singleton matching is a farsighted stable set if and only if the matching is stable.[6] Thus, stable matchings remain stable when agents become farsighted in marriage markets and in roommate problems. Do stable matchings survive when agents become farsighted in couples markets?

In this paper, we first provide a characterization of indirect dominance for matching markets with couples (Proposition 1) following the approach of Mauleon et al. (2011). A matching for a couples market indirectly dominates another matching if and only if no blocking coalition of the former is matched in the latter. We then show that, for couples markets, a singleton matching is a farsighted stable set if and only if the matching is stable. Thus, the property of stability of a matching is preserved when agents become farsighted.[7] From this characterization of singleton farsighted stable sets, it follows that whenever an algorithm returns a stable matching, the resulting matching is farsightedly stable in the sense that it is a very robust prediction because it indirectly dominates any other matching. Moreover, a farsighted stable set cannot contain exactly two matchings. However, other farsighted stable sets with multiple non-stable matchings can exist in couples markets with stable matchings. We also study farsighted stable sets in couples markets without stable matching, and we provide an example where a non-singleton farsighted stable exists and another one without a farsighted stable set. Then, we introduce for couples markets the DEM farsighted stable set of Herings et al. (2009, 2010) that replaces the internal stability condition in the definition of the farsighted stable set by deterrence of external deviations and minimality. Any farsighted stable set is a DEM farsighted stable set and then our main result (Theorem 1) also characterizes the singleton DEM farsighted stable sets. Contrary to the farsighted stable set, the DEM farsighted stable set always provides some robust predictions for couples markets with or without stable matchings. Indeed, DEM farsighted stable sets satisfy deterrence of external deviations and external stability, two essential properties guaranteeing the robustness of the matchings in the DEM farsighted stable sets.

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[5] See Mauleon et al. (2014) for a study of the condition that eliminates the differences between a farsighted solution concept and its myopic counterpart in one-to-one matching problems.

[6] Farsighted stability has also been studied in the case of hedonic games (Diamantoudi and Xue, 2004), non-transferable utility games (Ray and Vohra 2015), and matching problems including school choice problems (Atay et al. 2022a) and priority-based matching problems (Atay et al. 2022b).

[7] An important stream in the literature on matching markets studies whether a decentralized process of successive blocking leads to a stable matching. Under certain conditions on preferences (weakly responsiveness in Klaus and Klijn (2007); and restricted complementarity in Tello (2023)), it is possible to reach a stable matching from any arbitrary matching by satisfying blocking coalitions.

The paper is organized as follows. In Sect. 2, we present the model of two-sided matching with couples. In Sect. 3, we introduce the notion of farsighted stable set. In Sect. 4, we characterize indirect dominance. In Sect. 5, we study couples markets with stable matchings and we characterize singleton farsighted stable sets. We also provide an example of market with stable matching with a non-singleton farsighted stable set. In Sect. 6, we study couples markets without stable matchings and we introduce the DEM farsighted stable set as an alternative to the non-existence of the farsighted stable set. Section 7 concludes.

2 Matching with couples

A couples market consists of a set of hospitals $H = \{h_1, \ldots, h_m\}$ and a set of students $S = \{s_1, \ldots, s_{2n}\}$ partitioned into a set of couples $C = \{c_1, \ldots, c_n\} = \{(s_1, s_2), \ldots, (s_{2n-1}, s_{2n})\}$. Let s be a generic student and h be a generic hospital.

Each hospital $h \in H$ has exactly one position to fill and has a strict, complete and transitive preference relation $\succeq_h$ over the set of students S and the prospect of having its position unfilled, denoted by $\varnothing$. Let $s \in S$. If $s \succ_h \varnothing$, then student s is *acceptable* to hospital h; if $\varnothing \succ_h s$, then student s is *unacceptable* to hospital h. Hospital h's preferences can be represented by a strict ordering of the elements in $S \cup \{\varnothing\}$; for instance, $P(h) = s_1, s_3, \varnothing, ...$ indicates that hospital h prefers student s_1 to s_3, and considers the other students to be unacceptable. The preferences of all hospitals are denoted by $P_H = \{P_h\}_{h \in H}$.

Each couple $c \in C$ has a strict, complete and transitive preference relation $\succeq_c$ over all combinations of ordered pairs of different hospitals and the prospect of being unemployed. Couple c's preferences can be represented by a strict ordering of the elements in $\mathcal{H} := [H \cup \{u\} \times H \cup \{u\}] \backslash \{(h, h) | h \in H\}$ where u denote the prospect of being unemployed for one of the members of the couple. A generic element of $\mathcal{H}$ is denoted by (h_p, h_q), where h_p and h_q indicate a hospital or being unemployed. For instance, $P(c) = (h_1, h_3), (h_2, h_4), (h_3, u)$, etc., indicates that couple $c = (s_1, s_2)$ prefers s_1 and s_2 being matched to h_1 and h_3, respectively, to being matched to h_2 and h_4, respectively, and so on. A pair of hospitals $(h_p, h_q) \in \mathcal{H}$ are *acceptable* for c if $(h_p, h_q) \succ_c (u, u)$.[8] The preferences of all couples are denoted by $P_C = \{P_c\}_{c \in C}$. Let $P = (P_H, P_C)$ denote a couples market.[9] Let $\mathcal{P}$ be the set of all couples markets.

A *matching* μ for a couples market P is a function $\mu : S \cup H \to S \cup H \cup \{u, \varnothing\}$ such that:

(i) for any $s \in S$, $\mu(s) \in H \cup \{u\}$,
(ii) for any $h \in H$, $\mu(h) \in S \cup \{\varnothing\}$,
(iii) for any $s \in S$ and $h \in H$, $\mu(\mu(s)) = s$ and $\mu(\mu(h)) = h$.

[8] Whenever we use the strict part $\succ$ of a preference relation, we assume that we compare different elements in $S \cup \{\varnothing\}$ or $\mathcal{H}$.

[9] By simplicity, we assume that all students are members of a couple. However, we could easily add single students into the model since a single student corresponds to a couple where one of the members finds no hospital acceptable.

With some abuse of notation, we define $\mu(c) = (\mu(s_1), \mu(s_2)) \in \mathcal{H}$ for $c = (s_1, s_2)$. The set of matchings is denoted by $\mathcal{M}$. A student s and a hospital h are called *mates* if $\mu(s) = h$, or equivalently $\mu(h) = s$.

A hospital h is a *one-sided blocking coalition* to $\mu \in \mathcal{M}$ if h is matched with an unacceptable student in μ; i.e. $\varnothing \succ_h \mu(h)$. A couple c is a *one-sided blocking coalition* to $\mu \in \mathcal{M}$ if c is better off by unmatching one or both partners of the couple, i.e., $c = (s_1, s_2) \in C$ is a one-sided blocking coalition if either $\mu(s_1, s_2) \prec_c (u, u)$, $\mu(s_1, s_2) \prec_c (\mu(s_1), u)$ or $\mu(s_1, s_2) \prec_c (u, \mu(s_2))$. A matching $\mu \in \mathcal{M}$ is *individually rational* if there is no one-sided blocking coalition at μ.

We next define the two types of two-sided blocking coalitions. The first one involves both members of the couple and two hospitals, while the second one involves one hospital and one couple where the matching of the other member of the couple stays the same.

A couple $c = (s_1, s_2) \in C$ and two hospitals $h_1, h_2 \in H$ with $h_1 \neq h_2$ form a *two-sided blocking coalition* to $\mu \in \mathcal{M}$ if the couple c strictly prefers (h_1, h_2) to their mates at μ and $h_1, h_2 \in H$ with $h_1 \neq h_2$ strictly prefer respectively s_1 and s_2 rather than their mates at μ. Formally, $\{h_1, h_2, c = (s_1, s_2)\}$ are a two-sided blocking coalition if: (i) $(h_1, h_2) \succ_c \mu(s_1, s_2)$, (ii) $s_1 \succ_{h_1} \mu(h_1)$ and (iii) $s_2 \succ_{h_2} \mu(h_2)$.

A hospital $h_1 \in H$ and a couple $c = (s_1, s_2) \in C$ form a *two-sided blocking coalition* to $\mu \in \mathcal{M}$ if the couple strictly prefers the new pair of hospitals containing h_1 to her match under μ and h_1 strictly prefers $s_1 \in c$ to his mate at μ; i.e., if the following conditions hold: (i) $(h_1, \mu(s_2)) \succ_c \mu(s_1, s_2)$ and (ii) $s_1 \succ_{h_1} \mu(h_1)$.

A matching $\mu \in \mathcal{M}$ is *stable* if it is not blocked by any one-sided or two-sided blocking coalition. An alternative way to define a stable matching is by means of the direct dominance relation.

Given a matching $\mu \in \mathcal{M}$, a coalition $T \in H \cup C$ can enforce a matching $\mu' \in \mathcal{M}$ over μ if any match in μ' that does not exist in μ is between agents in T. Moreover, if a match in μ does not exist in μ', then one of the two agents involved in that match belongs to T. The next definition formalizes these ideas.

Definition 1 Given a matching μ, a coalition $T \in H \cup C$ can enforce a matching μ' over μ if the following conditions hold: For every $c \in C$ and $h \in H$, (i) $\mu'(h) \in c$ and $\mu'(h) \neq \mu(h)$ implies $\{c, h\} \subseteq T$ and (ii) $\mu(h) \in c$ and $\mu'(h) = \varnothing$ implies $\{c, h\} \cap T \neq \emptyset$.

Using this notion, we can describe the notion of direct dominance. We extend each agent's preference to the set of matchings in the following way. We say that agent i prefers μ' to μ if and only if agent i prefers her mate at μ' to her mate at μ, $\mu'(i) \succ_i \mu(i)$. Abusing notation, we write this as $\mu' \succ_i \mu$.

Definition 2 A matching μ is directly dominated by μ', or $\mu' > \mu$, if there exists a coalition $T \in H \cup C$ that can enforce μ' over μ such that $\mu' \succ_i \mu$ for all $i \in T$.

A matching $\mu \in \mathcal{M}$ is stable if it is not directly dominated by any other matching.

Given a matching $\mu \in \mathcal{M}$ with student $s \in S$ assigned to hospital $h \in H$, so $\mu(s) = h$, the matching μ' that is identical to μ, except that the match between s and h has been destroyed by either s or h (i.e., $\mu'(h) = \varnothing$ and $\mu'(s) = u$), is denoted

$\mu' = \mu - (s, h)$. Given a matching $\mu \in \mathcal{M}$ such that $s \in S$ and $h \in H$ are not matched to one another, the matching μ' that is identical to μ, except that the pair (s, h) has formed at μ' and their partners at μ (i.e., $\mu(s)$ and $\mu(h)$) become unmatched at μ', is denoted by $\mu' = \mu + (s, h)$.

3 Farsighted stable sets

The concept of stable matching is a myopic notion since the agents do not anticipate that individual and coalitional deviations could be followed by subsequent deviations. This concept and other concepts based on the direct dominance relation neglect the destabilizing effect of indirect dominance relations as introduced by Harsanyi (1974) and Chwe (1994). Indirect dominance captures the idea that farsighted agents can anticipate the actions of other coalitions and consider the end matching that their deviations may lead to.

Definition 3 A matching μ is indirectly dominated by μ', or $\mu' \gg \mu$, if there exists a sequence of matchings $\mu^0, ..., \mu^K$ (with $\mu^0 = \mu$ and $\mu^K = \mu'$) and a sequence of coalitions $T^0, ..., T^{K-1} \subseteq H \cup C$ such that for any $k \in 0, ..., K-1$, the following conditions hold:

1. Coalition T^k can enforce the matching μ^{k+1} over μ^k,
2. For all $i \in T^k$, $\mu' \succ_i \mu^k$.

The indirect dominance relation is denoted by $\ll$. Obviously, if $\mu' > \mu$, then $\mu' \gg \mu$. Based on the indirect dominance relation, the farsighted stable set (see Chwe (1994) and Mauleon et al. (2011)) is defined as the set of matchings satisfying internal and external stability conditions.

Definition 4 Let $P \in \mathcal{P}$ be a couples market. A set of matchings $V \subseteq \mathcal{M}$ is a farsighted stable set if it satisfies the following conditions:

(i) For every $\mu \in V$, there is no $\mu' \in V$ such that $\mu' \gg \mu$,
(ii) For every $\mu \notin V$, there is $\mu' \in V$ such that $\mu' \gg \mu$.

Condition (i) in Definition 4 is the internal stability (IS) condition: no matching inside the set is indirectly dominated by a matching belonging to the set. Condition (ii) is the external stability (ES) condition establishing that any matching outside the set is indirectly dominated by some matching belonging to the set.

Example 1 (Klaus and Klijn 2007) Consider a couples market with $H = \{h_1, h_2, h_3\}$ and $C = \{(s_1, s_2), (s_3, s_4)\}$. Hospitals' and couples' preferences are as follows:

Hospitals			Couples	
P_{h_1}	P_{h_2}	P_{h_3}	$P_{(s_1,s_2)}$	$P_{(s_3,s_4)}$
s_2	s_2	s_2	(h_3, h_1)	(h_2, h_3)
s_1	s_3	s_4	(h_2, h_3)	
	s_1	s_1	(h_1, h_2)	

There exist four individually rational matchings: $\mu_1 = \{(s_1, h_3), (s_2, h_1), (s_3, u), (s_4, u)\}$, $\mu_2 = \{(s_1, h_1), (s_2, h_2), (s_3, u), (s_4, u)\}$, $\mu_3 = \{(s_1, u), (s_2, u), (s_3, h_2), (s_4, h_3)\}$ and $\mu_4 = \{(s_1, h_2), (s_2, h_3), (s_3, u), (s_4, u)\}$. At μ_1, the coalition $\{h_2, h_3, (s_3, s_4)\}$ would be better off in μ_3 and then form a two-sided blocking coalition of μ_1. Since the coalition $\{h_2, h_3, (s_3, s_4)\}$ can enforce μ_3 from μ_1, we have that $\mu_3 > \mu_1$. At μ_3, the coalition $\{h_1, h_2, (s_1, s_2)\}$ form a two-sided blocking coalition of μ_3 and can enforce μ_2 so that $\mu_2 > \mu_3$.[10] From μ_2, the coalition $\{h_3, h_1, (s_1, s_2)\}$ form a two-sided blocking coalition of μ_2 and can enforce μ_1 so that $\mu_1 > \mu_2$. Thus, we have that $\mu_1 > \mu_2 > \mu_3 > \mu_1 \not< \mu_2 \not< \mu_3 \not< \mu_1$.

In this example, μ_4 is the unique stable matching. However, μ_4 does not directly dominate the three other individually rational matchings. But, starting at μ_1, looking forward to μ_4, h_3 will destroy its match with s_1 reaching the matching $\mu_1 - (s_1, h_3)$. After the deviation of h_3, the couple (s_1, s_2) is matched with (u, h_1), which is unacceptable for them. At $\mu_1 - (s_1, h_3)$, the coalition $\{h_2, h_3, (s_1, s_2)\}$ can enforce and will be better off at μ_4. Hence, the sequence of matchings μ^0, μ^1, μ^2 (where $\mu^0 = \mu_1$, $\mu^1 = \mu^0 - (s_1, h_3)$, and $\mu^2 = \mu^1 + \{h_2, h_3, (s_1, s_2)\} = \mu_4$) and the coalitions T^0, T^1 with $T^0 = \{h_3\}$ and $T^1 = \{h_2, h_3, (s_1, s_2)\}$ are such that T^0 can enforce μ^1 over μ^0 and coalition T^1 can enforce μ^2 over μ^1. Moreover, $\mu^2 \succ \mu^0$ for T^0 and $\mu^2 \succ \mu^1$ for $\{h_2, h_3, (s_1, s_2)\}$. Hence, we have that $\mu^2 = \mu_4 \gg \mu_1 = \mu^0$. Similar arguments can be used to show that μ_4 indirectly dominates μ_2, μ_3 as well as any other matching $\mu \neq \mu_4$. Therefore, $\{\mu_4\}$ is a farsighted stable set.[11]

Since direct dominance implies indirect dominance, we also have that $\mu_1 \gg \mu_2 \gg \mu_3 \gg \mu_1$. On the contrary, starting at μ_3 no agent has incentive to deviate looking forward to μ_1. Indeed, all agents matched at μ_3 formed a two-sided blocking coalition to μ_1. So, $\mu_3 \not\ll \mu_1$. Similar arguments can be used to show that $\mu_1 \not\ll \mu_2 \not\ll \mu_3 \not\ll \mu_1$. Thus, none of these three individually rational matchings form a singleton farsighted stable set. Since any not individually rational matching does not indirectly dominate the individually rational matchings, we have that $\{\mu_4\}$ is the unique singleton farsighted stable set. Farsighted stable sets that contain two (or more) matchings do not exist. Notice that, in order to satisfy external stability, the set should at least contain two of the three individually rational matchings. But then, since there exists no two individually rational matchings that do not indirectly dominate one another, internal stability will never be satisfied in any set containing two of the three individually rational matchings. Therefore, the unique stable matching of this couples market when agents are myopic is also the only farsighted stable set.

In this example, we have shown that a singleton stable matching is the unique farsighted stable set. Mauleon et al. (2011) characterized farsighted stable sets for marriage markets and showed that a set of matchings is a farsighted stable set if and only if it is a singleton stable matching. Klaus et al. (2011) showed that in roommate problems a singleton is a farsighted stable set if and only if the matching is stable. In Sect. 5 we study whether these results hold for couples markets. Furthermore, we

[10] Notice that if h_3 would have been able to anticipate the next deviation from μ_3 to μ_2, it would not have deviated from μ_1 to μ_3. Thus, $\mu_1 \not\ll \mu_2$.

[11] Klaus and Klijn (2007) used this example to show that a (myopic) path to a stable matching obtained from satisfying blocking coalitions does not always exist.

investigate the existence of non-singleton farsighted stable sets in couples markets with stable matchings.

4 Characterizing indirect dominance

Following Mauleon et al. (2011), we characterize indirect dominance for couples markets (Proposition 1): a matching indirectly dominates another matching if and only if no blocking coalition of the former is matched in the latter. To obtain this characterization, Lemma 1 first provides a necessary condition for indirect dominance: if a matching μ indirectly dominates another matching μ', then there does not exist in μ' a one-sided or a two-sided blocking coalition of μ. Lemma 2 provides a sufficient condition for indirect dominance stating that an individually rational matching μ indirectly dominates another matching μ' if there does not exist any two-sided blocking coalition of μ matched in μ'. Then, Proposition 1 provides the characterization of indirect dominance by covering the case that was uncovered in Lemma 2; i.e., the case where there is no two-sided blocking coalition of μ matched in μ' but one of the hospitals matched in μ' strictly prefers μ and starts the deviation from μ' to μ.[12]

The next lemma shows that if a matching μ indirectly dominates another matching μ', then there is no one-sided blocking coalition of μ matched at μ' and there is no two-sided blocking coalition of μ matched at μ'.

Lemma 1 *If $\mu \gg \mu'$, then there is no hospital $h \in H$ such that $\mu'(h) = \varnothing \succ_h \mu(h)$, there is no couple $c = (s_1, s_2) \in C$ such that $\mu(s_1, s_2) \prec_c \mu'(s_1, s_2)$ with $\mu'(s_1, s_2) \in \{(u, u), (\mu(s_1), u), (u, \mu(s_2))\}$, and there does not exist $h \in H$ and $c \in C$ with $\mu'(h) \in c$ such that both c and h strictly prefer μ' to μ.*

Proof Suppose on the contrary that $\mu \gg \mu'$ and that there exists a two-sided blocking coalition $\{h, h', c = (s, s')\}$ such that $\mu'(c) = (h, h')$, $(h, h') \succ_c \mu(s, s')$, $s \succ_h \mu(h)$ and $s' \succeq_{h'} \mu(h')$. For μ to indirectly dominate μ', it must be that either h, or h' or c get unmatched along the path from μ' to μ. But they all prefer μ' to μ, and then they will never unmatch. Hence $\mu \not\gg \mu'$, a contradiction. The proof for a one-sided blocking coalition is similar. Since the member(s) of this one-sided coalition prefer to be unmatched than being matched at μ, they will never match along the path from μ' to μ. Hence $\mu \not\gg \mu'$, a contradiction. □

From Lemma 1, it follows that μ should be individually rational in order to dominate μ'.

Lemma 2 *Consider any two matchings $\mu, \mu' \in \mathcal{M}$ such that μ is individually rational. Then $\mu \gg \mu'$ if there does not exist $h \in H$ and $c \in C$ with $\mu'(h) \in c$ such that both c and h strictly prefer μ' to μ.*

Proof Let $B(\mu', \mu)$ be the set of hospitals and couples that strictly prefer μ to μ' and let $I(\mu', \mu)$ be the set of couples and hospitals who are indifferent between μ and

[12] Notice that the characterization of indirect dominance for couple markets is similar to the one obtained by Mauleon et al. (2011) for one-to-one matching problems without couples. The only difference is that here two types of one- and two-sided blocking coalitions need to be considered.

μ'. We prove the lemma showing that if the above condition is satisfied then $\mu \gg \mu'$ because we can construct a sequence of matchings and blocking coalitions starting at μ' and leading to μ that satisfies Definition 3.[13]

Construct the following sequence of matchings from μ' to μ: $\mu^0 = \mu'$, $\mu^1 = \mu' - B(\mu', \mu)$, and $\mu^2 = \mu$. Consider the following sequence of coalitions $T^0 = B(\mu', \mu)$ and $T^1 = B(\mu', \mu) \cup \{i \in H \cup C \backslash I(\mu', \mu) | \mu'(i) \subseteq B(\mu', \mu)\}$. Notice that coalition T^0 can enforce μ^1 over μ^0 and, by definition of $B(\mu', \mu)$, we have that $\mu^2 \succ \mu^0$ for T^0.

We show next that each hospital and student of T^1 (except the agents having the same matching in μ and μ') are unmatched in μ^1 and since $\mu = \mu^2$ is individually rational, they prefer μ^2 to μ^1.

Given the above condition, we know that if one member of the couple c was unemployed in μ' but not in μ, either the couple or the hospital matched with the other member of the couple would belong to $B(\mu', \mu)$. Then, both the couple c and the hospitals in $\mu(c)$ will all be unmatched in μ^1. Given the condition of Lemma 2, it holds that if $c = (s_1, s_2) \in C$ and $(h_1, h_2) \in \mathcal{H}$ are such that $\mu'(c) = (h_1, h_2) \neq \mu(c)$, either (i) $\mu' \succ_c \mu$ and $\mu \succeq_{h_1,h_2} \mu'$, or (ii) $\mu \succ_c \mu'$. In case (ii), since $c \in B(\mu', \mu)$, h_1 and h_2 will also be unmatched in μ^1. In case (i), without loss of generality, $h_1 \in B(\mu', \mu)$ and either $h_2 \in B(\mu', \mu)$ or $\mu'(h_2) = \mu(h_2) = s_2$.[14] If $h_2 \in B(\mu', \mu)$, c, h_1 and h_2 will be unmatched in μ^1 and since μ is individually rational it holds that they all strictly prefer μ^2 to μ^1. If $\mu'(h_2) = \mu(h_2) = s_2$, h_1 and s_1 will still be unmatched in μ^1. In addition, if $\mu(s_1) \neq u$, since μ is individually rational, it holds that $(\mu(s_1), h_2) \succ_c (\emptyset, h_2)$. Otherwise, if the couple prefers that s_1 becomes unemployed, c would be a one-sided blocking coalition of μ.[15] Thus, in all of these cases, c and h_1 (and h_2 if it is not indifferent between μ and μ') strictly prefer μ^2 to μ^1.

Furthermore, since all the agents who changed their matching between μ' and μ are either included in $B(\mu', \mu)$ or matched in μ' with a member of $B(\mu', \mu)$ (and then unmatched in μ^1), we have that μ^2 is enforceable from μ^1 by T^1. Hence, $\mu \gg \mu'$. □

Remark 1 Lemma 2 is a sufficient condition for indirect dominance. Lemma 1 is a necessary condition for indirect dominance that is less restrictive. Under the condition of Lemma 2, there does not exist any two-sided blocking coalition of μ matched in μ'. Moreover, since μ is individually rational, there exists no one-sided blocking coalition of μ matched at μ'. However, even if there exists no blocking coalition of μ matched at μ', there could still exist $c \in C$ and $h_1, h_2 \in H$ such that $\mu'(c) = (h_1, h_2)$ with $\mu' \succ_{c,h_1} \mu$ and $\mu \succ_{h_2} \mu'$.

The next proposition covers this case and provides a condition for indirect dominance that is necessary and sufficient.

Proposition 1 *Consider any two matchings $\mu, \mu' \in \mathcal{M}$ such that μ is individually rational. Then $\mu \gg \mu'$ if and only if there does not exist any blocking coalition of μ matched in μ'.*

13 Our proof follows the proof of Lemma 1 from Mauleon et al. (2011).

14 The proof would be the same if we took $h_2 \in B(\mu', \mu)$ and either $h_1 \in B(\mu', \mu)$ or $\mu'(h_1) = \mu(h_1)$.

15 If $\mu(s_1) = u$, $\mu(c)$ will already be formed in μ^1.

Proof The "*only if*" part follows from Lemma 1.

Lemma 2 guarantees the "*if*" part for the case where there exists no $h \in H$ and $c \in C$ with $\mu'(h) \in c$, such that both c and h strictly prefer μ' to μ. However, Lemma 2 does not cover the case where there exist $c \in C$, $h_1, h_2 \in H$ such that $\mu'(c) = (h_1, h_2)$, $\mu' \succ_{c,h_1} \mu$ and $\mu \succ_{h_2} \mu'$.

Since $h_2 \in B(\mu', \mu)$, h_2 will join the first coalition T^0 that initiates the sequence of matchings leading from μ' to μ. After the deviation of T^0 from $\mu' = \mu^0$ to μ^1, we have $\mu^1(c) = (h_1, u)$. At μ^1, it must hold that $\mu(c) \succ_c \mu^1(c)$ and/or $\mu(h_1) \succ_{h_1} \mu^1(h_1)$. Otherwise, $\{c, h_1\}$ will form a blocking coalition of μ matched at μ', contradicting the hypothesis of non-existence of blocking coalitions of μ matched at μ'. Hence, from μ^1, either c, or h_1, or both, will become unmatched moving to μ^2. Finally, from μ^2, since all the agents who changed their matching between μ' and μ are either included in $B(\mu', \mu)$, or included in $B(\mu_1, \mu)$, or matched in μ' with a member of $B(\mu', \mu)$ (and then unmatched in μ^1 or in μ^2), we have that $\mu^3 = \mu$ is enforceable from μ^2 by $T^2 = B(\mu', \mu) \cup B(\mu_1, \mu) \cup \{i \in H \cup C \backslash I(\mu', \mu) | \mu'(i) \subseteq B(\mu', \mu)\}$ and such that $\mu^3 \succ_{T^2} \mu^2$. Hence, $\mu \gg \mu'$. □

5 Markets with stable matchings

In this section, we study couples with stable matchings. First, using Proposition 1 we characterize singleton farsighted stable sets (Theorem 1). Second, we show that although there are no farsighted stable sets of cardinality two (Lemma 3), there might exist farsighted stable sets containing several matchings (Example 2).

Theorem 1 *A singleton $V = \{\mu\}$ is a farsighted stable set if and only if μ is a stable matching.*

Proof Since μ is a stable matching, there exists no blocking coalitions of μ in any $\mu' \in \mathcal{M}$. By Proposition 1, this condition is satisfied if and only if $\mu \gg \mu'$ for any $\mu' \in \mathcal{M}$, which is the definition of a singleton farsighted stable set.

Let $\{\mu\}$ be a farsighted stable set. By the external stability condition of Definition 4 and by Lemma 1, there exists no blocking coalition at μ matched in any $\mu' \in \mathcal{M}$. Hence, μ is a stable matching. □

Theorem 1 characterizes singleton farsighted stable sets: a singleton matching is a farsighted stable set if and only if the matching is stable. Thus, the property of stability of a matching is preserved when agents become farsighted since a stable matching indirectly dominates any other matching. We next investigate the existence of non-singleton farsighted stable sets. In marriage markets, Mauleon et al. (2011) showed that the only farsighted stable sets were the singletons consisting of a stable matching. Contrary to marriage markets, we show that farsighted stable sets with several elements can exist for couples markets. First, using Lemma 1, we can show that no farsighted stable set can be composed of exactly two elements.

Lemma 3 *There does not exist a farsighted stable set of cardinality two.*

Proof Suppose on the contrary that there exists a farsighted stable $V = \{\mu_1, \mu_2\}$, with $\mu_1 \neq \mu_2$. Notice first that μ_1 and μ_2 should not be stable. Otherwise, since a stable matching indirectly dominates any other matching, the internal stability condition of Definition 4 would be violated. Second, both μ_1 and μ_2 should be individually rational. Otherwise, if μ_1 is not individually rational, it would not indirectly dominate the matchings respecting one-sided blocking coalitions of μ_1. In order to satisfy the external stability condition of Definition 4, μ_2 must then indirectly dominate the matchings respecting one-sided blocking coalitions of μ_1. But in this case μ_2 would also indirectly dominate μ_1 violating the internal stability condition of Definition 4.

Since both μ_1 and μ_2 are unstable individually rational matchings belonging to $V = \{\mu_1, \mu_2\}$, we can construct a matching μ_1' from μ_1 enforced by the two-sided blocking coalitions of μ_1 matched in μ_2. Otherwise, if there does not exist any blocking coalition of μ_1 matched at μ_2, $\mu_1 \gg \mu_2$, violating the internal stability condition of Definition 4.[16] By Lemma 1 and the construction of μ_1', we have that $\mu_1 \not\gg \mu_1'$ because at least one blocking coalition of μ_1 is matched in μ_1'. By external stability of V, we have that $\mu_2 \gg \mu_1'$. Then, there does not exist a two-sided blocking coalition of μ_2 matched in μ_1. Otherwise, the two-sided blocking coalition would also be matched in μ_1'. Since the two-sided blocking coalition is worse off in μ_2 compared to μ_1', their members would never join any deviation leading to μ_2, contradicting the fact that $\mu_2 \gg \mu_1'$. Thus, by Lemma 1, this implies that $\mu_2 \gg \mu_1$, which contradicts the internal stability of V. □

The next example shows that there might exist farsighted stable sets with more than two elements for couples markets with stable matchings.

Example 2 Consider a matching with couples problem with $H = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $C = \{(s_1, s_2), (s_3, s_4), (s_5, s_6), (s_7, s_8), (s_9, s_{10})\}$. Hospitals and couples' preferences are as follows.

Hospitals						Couples				
P_{h_1}	P_{h_2}	P_{h_3}	P_{h_4}	P_{h_5}	P_{h_6}	$P_{(s_1,s_2)}$	$P_{(s_3,s_4)}$	$P_{(s_5,s_6)}$	$P_{(s_7,s_8)}$	$P_{(s_9,s_{10})}$
s_4	s_2	s_5	s_1	s_8	s_2	$P_{(s_1,s_2)}$	$P_{(s_3,s_4)}$	$P_{(s_5,s_6)}$	$P_{(s_7,s_8)}$	$P_{(s_9,s_{10})}$
s_8	s_6	s_7	s_3	s_6	s_5	(h_1, h_2)	(h_4, h_1)	(h_6, h_5)	(h_5, h_6)	(h_2, u)
s_1	s_9	s_3	s_9	s_7	s_8	(h_4, h_6)	(h_3, u)	(h_3, u)	(h_3, h_5)	(h_4, u)
s_{10}								(u, h_2)	(h_5, h_1)	(u, h_1)

There exists a unique stable matching $\mu = \{(s_1, h_4), (s_2, h_6), (s_3, u), (s_4, u), (s_5, h_3), (s_6, u), (s_7, h_5), (s_8, h_1), (s_9, h_2), (s_{10}, u)\}$. Then, Theorem 1 implies that the singleton set $V = \{\mu\}$ is a farsighted stable set. Consider now the following three matchings:

[16] Note that μ_1' is not necessarily individually rational.

$$\mu_1 = \{(s_1, h_1), (s_2, h_2), (s_3, h_3), (s_4, u), (s_5, h_6), (s_6, h_5), (s_7, u), (s_8, u), (s_9, h_4), (s_{10}, u)\},$$
$$\mu_2 = \{(s_1, h_4), (s_2, h_6), (s_3, u), (s_4, u), (s_5, u), (s_6, h_2), (s_7, h_3), (s_8, h_5), (s_9, u), (s_{10}, h_1)\},$$
$$\mu_3 = \{(s_1, u), (s_2, u), (s_3, h_4), (s_4, h_1), (s_5, h_3), (s_6, u), (s_7, h_5), (s_8, h_6), (s_9, h_2), (s_{10}, u)\}.$$

We show now that the set $V' = \{\mu_1, \mu_2, \mu_3\}$ is another farsighted stable set.

The matching μ_2 is blocked by the coalition $\{h_1, h_2, (s_1, s_2)\}$. Moreover, the couple (s_1, s_2) and hospitals h_1 and h_2 are matched under the matching μ_1. So, from Lemma 1, $\mu_2 \not\gg \mu_1$. On the other hand, the matching μ_1 is blocked by the coalition $\{h_3, h_5, (s_7, s_8)\}$. Moreover, the couple (s_7, s_8) and hospitals h_3 and h_5 are matched under the matching μ_2. Hence, from Lemma 1, $\mu_1 \not\gg \mu_2$.

The matching μ_3 is blocked by the coalition $\{h_5, h_6, (s_5, s_6)\}$. Moreover, student s_5 and hospital h_6 and student s_6 and h_5 are matched under the matching μ_1. So, from Lemma 1, $\mu_3 \not\gg \mu_1$. On the other hand, the matching μ_1 is blocked by the coalition $\{h_4, h_1, (s_3, s_4)\}$. Moreover, the couple (s_3, s_4) and hospitals h_4 and h_1 are matched under the matching μ_3. Hence, from Lemma 1, $\mu_1 \not\gg \mu_3$.

The matching μ_3 is blocked by the coalition $\{h_4, h_6, (s_1, s_2)\}$. Moreover, the couple (s_1, s_2) and hospitals h_4, h_6 are matched under the matching μ_2. So, from Lemma 1, $\mu_3 \not\gg \mu_2$. On the other hand, the matching μ_2 is blocked by the coalition $\{h_3, (s_5, s_6)\}$. Moreover, the couple (s_5, s_6) and hospitals u, h_3 are matched under the matching μ_3. Hence, from Lemma 1, $\mu_2 \not\gg \mu_3$.

Since there is no matching in the set V' that indirectly dominates another matching in V', the set $V' = \{\mu_1, \mu_2, \mu_3\}$ satisfies the internal stability condition of Definition 4.

In addition, the stable matching μ is indirectly dominated by μ_1. The sequence of matchings $\mu^0 = \mu$, $\mu^1 = \mu^0 - \{h_5, h_2, (s_1, s_2), (s_5, s_6)\}$, $\mu^2 = \mu^1 - \{(s_7, s_8)\}$, $\mu^3 = \mu^2 + \{h_1, h_2, h_3, h_4, h_5, h_6, (s_1, s_2), (s_3, s_4), (s_5, s_6), (s_9, s_{10})\} = \mu_1$ are enforceable respectively by the coalitions $T^0 = \{h_5, h_2, (s_1, s_2), (s_5, s_6)\}$, $T^1 = \{(s_7, s_8)\}$, $T^2 = \{h_1, h_2, h_3, h_4, h_5, h_6, (s_1, s_2), (s_3, s_4), (s_5, s_6), (s_9, s_{10})\}$. Notice that, at each step, the members of T^k, $k = \{0, 1, 2\}$, are better off in μ_1 than in μ^k. Thus, $\mu_1 \gg \mu$. It can easily be checked that all other matchings are indirectly dominated by some matching inside V'. Thus, V' also satisfies the external stability condition in Definition 4. Therefore, the set $V' = \{\mu_1, \mu_2, \mu_3\}$ is a farsighted stable set.

This example shows that couples markets can exhibit a farsighted stable set containing unstable matchings, which is not the case for marriage markets. See Theorem 2 in Mauleon et al. (2011). The presence of couples seeking for positions in the same labor market and the fact that members of a couple do not only care about their own assignment but also about their partner's assignment generate a complementarity in the couple's preferences that could explain the differences between our results and the ones for marriage markets. Notice that Example 2 exhibits not only the complementarity that both students in a couple want to be employed, but also selectivity (students only want to be employed as a couple at specific hospital pairs) and a complementarity between students and hospitals, e.g., couple (s_9, s_{10}) prefers s_9 being matched to h_4 (and s_{10} being unemployed) to s_{10} being matched with h_1 (and s_9 being unemployed).

6 Markets without stable matchings

In this section we study the farsighted stable set in couples markets without stable matchings. From Theorem 1, we deduce that in these markets without stable matchings there is no singleton farsighted stable set. We provide two examples to show that the existence of farsighted stable sets is not guaranteed for couples markets without stable matchings. The next example provides a couples market without stable matchings but with a farsighted stable set containing several matchings.

Example 3 Consider the couples market derived from Example 2 by removing the acceptable pair of hospitals (h_5, h_1) for the couple (s_7, s_8) and the acceptable doctor s_8 for hospital h_1. Hospitals and couples' preferences are as follows.

Hospitals						Couples				
P_{h_1}	P_{h_2}	P_{h_3}	P_{h_4}	P_{h_5}	P_{h_6}	$P_{(s_1,s_2)}$	$P_{(s_3,s_4)}$	$P_{(s_5,s_6)}$	$P_{(s_7,s_8)}$	$P_{(s_9,s_{10})}$
s_4	s_2	s_5	s_1	s_8	s_2	(h_1, h_2)	(h_4, h_1)	(h_6, h_5)	(h_5, h_6)	(h_2, u)
s_1	s_6	s_7	s_3	s_6	s_5	(h_4, h_6)	(h_3, u)	(h_3, u)	(h_3, h_5)	(h_4, u)
s_{10}	s_9	s_3	s_9	s_7	s_8			(u, h_2)		(u, h_1)

Notice that there does not exist a stable matching. However, the set of matchings $V = \{\mu_1, \mu_2, \mu_3\}$ is still a farsighted stable set as we can easily check that it respects the internal and external stability conditions from Definition 4.[17]

The next example provides a couples market without a farsighted stable set.

Example 4 (Roth 2008) Consider a couples market with $H = \{h_1, h_2\}$ and $C = \{(s_1, s_2), (s_3, s_4)\}$. Hospitals and couples' preferences are as follows.

Hospitals		Couples	
P_{h_1}	P_{h_2}	$P_{(s_1,s_2)}$	$P_{(s_3,s_4)}$
s_1	s_1	(h_1, h_2)	(h_1, u)
s_3	s_3		(h_2, u)
s_2	s_2		

Roth (2008) showed that there does not exist a stable matching in this couples market. One can see that there are three individually rational matchings: $\mu_1 = \{(s_1, h_1), (s_2, h_2), (s_3, u), (s_4, u)\}$, $\mu_2 = \{(s_1, u), (s_2, u), (s_3, h_2), (s_4, u)\}$, and $\mu_3 = \{(s_1, u), (s_2, u), (s_3, h_1), (s_4, u)\}$. Notice that hospital h_1 and couple (s_3, s_4) are better off at μ_3 than at μ_2, and hence $\mu_3 > \mu_2$. Since hospital h_2 and couple (s_3, s_4) are better off at μ_2 than at μ_1, it holds that $\mu_2 > \mu_1$. Finally, since hospitals h_1, h_2 and couple (s_1, s_2) are better off at μ_1 than at μ_3, we have $\mu_1 > \mu_3$. Thus, there is no stable matching. Furthermore, since direct dominance implies indirect dominance, we have that $\mu_1 \gg \mu_3 \gg \mu_2 \gg \mu_1$. Then, any set V containing at least two matchings ($V \supseteq \{\mu_1, \mu_2\}$, $V \supseteq \{\mu_1, \mu_3\}$, and $V \supseteq \{\mu_2, \mu_3\}$) does not satisfy the internal

[17] This is because we have only removed some unacceptable agents from the preferences in Example 2 but the order of the preferences has not been modified.

stability condition from Definition 4 and is not a farsighted stable set. Moreover, any singleton set V is not a farsighted stable set since μ_3 does not indirectly dominate μ_1, μ_1 does not indirectly dominate μ_2, and μ_2 does not indirectly dominate μ_3. Then, the external stability condition from Definition 4 would be violated.

Thus, there are couples markets where the farsighted stable set does not exist. When agents are farsighted, is there any solution concept that always provides some prediction? Herings et al. (2009, 2010) propose the DEM farsighted stable set for network formation problems and coalition formation problems respectively.[18] Replacing the internal stability condition in the definition of the farsighted stable set by deterrence of external deviations and minimality, leads to a stability concept, the DEM farsighted stable set, that is always non-empty.

Definition 5 Let $P \in \mathcal{P}$ be a couples market. A set of matchings $V \subseteq \mathcal{M}$ is a DEM farsighted stable set if it satisfies:

(D) *Deterrence of external deviations*: For any $\mu \in V$, and any $\mu' \notin V$ that can be enforced from μ by coalition $T \in H \cup C$, there exists $\mu'' \in V$ such that $\mu'' \gg \mu'$ and $\mu'' \not\succ_T \mu$.

(E) *External stability*: For every $\mu \notin V$, there exists $\mu' \in V$ such that $\mu' \gg \mu$,

(M) *Minimality*: There does not exist $V' \subset V$ such that V' satisfies both (D) and (E).

Condition (D) in Definition 5 requires the deterrence of external deviations. It captures that any deviation to a matching outside V, is deterred by the threat of ending in μ'' that indirectly dominates μ'. Moreover μ'' belongs to V which makes μ'' a credible threat. Condition (E) in Definition 5 requires external stability and implies that the matchings within the set are robust to perturbations. Any matching outside of V is indirectly dominated by a matching in the set. Condition (E) implies that if a set of matchings is a DEM farsighted stable set, it is non-empty. Notice that the set $\mathcal{M}$ (trivially) satisfies Conditions (D) and (E) in Definition 5. This motivates the requirement of a minimality condition, namely Condition (M).

Contrary to farsighted stable sets, DEM farsighted stable sets always exist. Moreover, if V is a farsighted stable set, then V is a DEM farsighted stable set. See Herings et al. (2009, 2010). In Example 4 there is no farsighted stable set. However, the sets $V = \{\mu_1, \mu_2\}$, $V' = \{\mu_1, \mu_3\}$, and $V''\{\mu_2, \mu_3\}$ are DEM farsighted stable sets. For instance, $V = \{\mu_1, \mu_2\}$ satisfy external stability and deterrence of external deviations because the deviation from μ_2 to μ_3 of coalition $\{h_1, (s_3, s_4)\}$ is deterred by the subsequent deviation of coalition $\{h_1, h_2, (s_1, s_2)\}$ from μ_3 to μ_1 where the couple (s_3, s_4) is worse off than at μ_2.

The DEM farsighted stable set could be used to predict the matchings that are stable when agents are farsighted and the farsighted stable set does not exist. Since any farsighted stable set is a DEM farsighted stable set, our main result (Theorem 1) also characterizes the singleton DEM farsighted stable sets. Moreover, for couples

[18] The DEM farsighted stable set was initially called the pairwise farsightedly stable set (Herings et al. 2009) and farsighted stable set (Herings et al. 2010). To avoid the confusion between the notion of farsighted stable set studied in this paper and these two concepts, we follow Kimya (2023) and de Callataÿ et al. (2024) and call it DEM farsighted stable set.

markets with or without stable matchings, contrary to the farsighted stable set, the DEM farsighted stable set always provides some robust predictions. Indeed, DEM farsighted stable sets satisfy deterrence of external deviations and external stability, two essential properties guaranteeing the robustness of the matchings in the DEM farsighted stable sets. A characterization of the DEM farsighted stable set is out of the scope of the paper.

7 Concluding remarks

We adopt the notion of the farsighted stable set to determine which matchings are stable when agents are farsighted in matching markets with couples. We show that a singleton matching is a farsighted stable set if and only if the matching is stable. Thus, the property of stability of a matching is preserved when agents become farsighted in couples markets. However, other farsighted stable sets with multiple non-stable matchings can exist in couples markets with stable matchings. For markets without stable matching, we provide examples of markets with and without farsighted stable sets. For couples markets where the farsighted stable set does not exist, the DEM farsighted stable set could be used to predict the matchings that are stable when agents are farsighted.

The matching with couples problem is highly relevant in practice (e.g., the National Residency Match Program (NRMP)). Since the redesign of NRMP by Roth and Peranson (1999), the matching algorithm that is used in the program is a heuristic based on the incremental algorithm by Roth and Vande Vate (1990). However, Klaus et al. (2007) showed that this method may not find a stable matching, even if its existence is guaranteed. Thus, a deeper study of whether or not a matching obtained from an algorithm (e.g. the Roth-Peranson Algorithm) is farsightedly stable remains a challenging question. However, from the characterization of singleton farsighted stable sets (Theorem 1) it follows that whenever an algorithm returns a stable matching, the resulting matching is farsightedly stable in the sense that it is a very robust prediction because it indirectly dominates any other matching.

It has been assumed in the paper that each hospital has exactly one position to fill. If hospitals have more than one position to fill and couples are allowed to apply to a pair of positions at the same hospital, several stability concepts have been proposed (see Kojima et al. 2013; McDermid and Manlove 2010; Marx and Schlotter 2011; Biró et al. 2011). Studying to which specific stability concept our results generalize (if any) is not an easy question. However, our findings can be generalized straightforwardly to scenarios where hospitals have multiple positions, hospitals' preferences are responsive,[19] and no couple applies to a pair of positions at the same hospital.

The results obtained in the paper have been obtained for general preferences' domain. For this general domain, the characterization of farsighted stable sets in Mauleon et al. (2011) for marriage markets does not carry over to couples markets. Analyzing whether or not the characterization of Mauleon et al. (2011) holds for

[19] A hospital's preferences over groups of students are responsive if, for any two assignments that differ in only one student, it prefers the assignment containing the more preferred student with respect to their preferences over single students.

restricted domains (e.g., weak responsiveness as in Klaus and Klijn (2005) or master lists as in Ashlagi et al. (2014)) remains a challenging and interesting question.

Recently, the notion of the myopic-farsighted stable set has been proposed to study the interaction between myopic and farsighted agents in one-to-one matching markets. See Herings et al. (2020a, b) and Doğan and Ehlers (2023). Only when farsighted agents are present on one side of the market, the existence of myopic-farsighted stable sets is guaranteed. Moreover, the introduction of heterogeneous agents or of limited farsighted agents destabilize some stable matchings (the one obtained from the DA algorithm) and stabilize other matchings (the one obtained from the Top Trading Cycle algorithm) in school choice problems and priority-based matching problems (see Atay et al. 2022a, b). An interesting extension would be to study matching with couples when agents are heterogeneous in their degree of farsightedness.

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